CRITICAL RESEARCHES
ON
GENERAL ELECTRODYNAMICS

Introduction
and
First Part

By

WALTER RITZ

1908

Lucier-Fritzius-Toth
Translation
1980

Published
by

Robert S. Fritzius
305 Hillside Drive
Starkville MS
39759
U.S.A.

CONTENTS

[1]Page numbering in body of translation corresponds to that in Annales de Chimie et de Physique, 8e serie, t. XIII, (Fevrier 1908).

[2] Page numbering from Œuvres de Walther Ritz is shown in margins of text.

CRITICAL RESEARCHES ON GENERAL ELECTRODYNAMICS[1]

BY WALTER RITZ

INTRODUCTION

Electric and electrodynamic phenomena have acquired in the course of these last years more and more importance. They include Optics, the laws of radiation and the innumerable molecular phenomena associated with the presence of charged centers, ions and electrons. Finally, with the notion of electromagnetic mass, Mechanics itself seems obliged to become a chapter of General Electrodynamics. In the form given to it by H. A. Lorentz, Maxwell's theory would thus become the turning point towards a new conception of nature, where the laws of electrodyamics, considered as primary, would contain the laws of motion as special cases,
and would play the fundamental role in the physical theories which, un- 317
til now, have belonged to Mechanics.

Under these circumstances, it is plainly desirable to have a rigorous criticism of the foundations of this theory, to give it the degree of clarity and precision that Mechanics itself reached only recently after much controversy. It is in order to ask: which hypotheses are essential and can be deduced from observation, which others are logically useless or can be discarded without experience ceasing to be adequately represented, and finally, which are those which can be, and should be, rejected; a question which is asked principally in regard to
absolute motion. 318

[1]Translated (1980) from "Recherches Critiques Sur L'Électrodynamique Générale", Ann. de Chim. et de Phys., 8e serie, t. XIII (Fevrier 1908)

In the first part of his "Lessons on Electricity and Optics"[1], Poincaré devoted some classic pages to the criticism of the more or less distinctive theories of Maxwell himself and of Hertz; therefore I will concern myself only with the form that the theory took in the hands of Lorentz, a form that presents well known advantages. Some of his results can easily be extended to the other theories. Here again, I only have to recall, or to complete, the ideas put forward by Poincaré and more importantly by Lorentz who was well aware of the different aspects under which his theory could be interpreted.

In general, I set aside the phenomena of molecular order, dependent on the corpuscular theory of electricity: this fruitful concept is evidently independent, in large part, of ideas that we can develop about the mode of action of electric charges on one another 318
via the ether medium, which are more specifically the object of electrodynamic equations.

The result of these researches has not been favorable to the existing [field] theories. The discussions about the difficulties that they raise show that the difficulties have a common origin intimately linked to the concept of ether which is the basis of all these theories. We will see specifically that:

1. From a strictly logical point of view, the electric and magnetic forces, which, in appearance, play in the theory [3] a role so fundamental, are notions which we can eliminate entirely: they only contain in reality the relations of space and time: we thus return to the old elementary actions, with this sole difference that they are no longer instantaneous.

---

[1] H. POINCARE, Electricite et Optique: La lumiere et las theories electrodynamiques. 2 eme ed. Paris 1901

[3] 'Maxwell-Lorentz'

2. The theory [1] permits an infinite number of solutions, each satisfying all the conditions, but incompatible with experience and even leading for example to perpetual motion. To remove these solutions we must admit by hypothesis formulae for retarded potentials. These formulae introduce irreversibility in to electrodynamics whereas the general equations permit reversibility. I show that, contrary to accepted 319
ideas, that they [the formulae for retarded potentials] can't be deduced from a proper specialization of the initial state. They constitute a new hypothesis, making useless the partial differential equations. To clarify this hypothesis, it is necessary to distinguish the elementary actions; it is to renounce Maxwell's fundamental idea of rejecting them.

3. The notion of localization of energy in the ether is vague and allows many simple solutions.

4. The impossibility, 25 described by Maxwell, to reduce gravitation 319
to the same notions. That the negative energy involved would correspond to an unstable system, shows that these ideas do not have general applicability to the forces of nature.

5. Action and reaction are not equal, and this inequality, in the manner in which it is deduced from the introduction of absolute velocities, is contrary to experience.

6. Kaufmann's experiments on the electric and magnetic deviability of beta rays of radium don't prove that the mass of electrons is entirely of electromagnetic origin, and dependent on their absolute velocity, because on the first hand, nothing obliges us to believe, as in Lorentz's theory, that the forces are linear functions of velocity, (this may be true at small velocities), and that, on the other hand, one of Trouton and Noble's experiments shows that the expression of electromagnetic momentum

[1] Maxwell-Lorentz

as a function of velocity from which Abraham has deduced the one of electromagnetic mass is certainly inexact.

7. The theory of Maxwell and of Lorentz starts from a system of absolute coordinates, that is to say, independent of all motions of matter. In order to be in agreement with experimental results, which have always, in Optics and Electrodynamics, as well as in Mechanics, confirmed the principle of relative motions, we are obliged, then, to eliminate this absolute system by hypotheses of little credibility, thus eliminating the notion of solid bodies, and the concept of the invariability of ponderable masses. It will be necessary also, to change the principles of Kinematics, to consider the rule of the velocity parallelogram just as a first approximation, valid at small speeds, and to make time and simultaniety completely relative notions. 320

It would be regrettable, for the economy of our thinking if we had 320
to live with all the complications listed above. I think, that instead of Kinematics, it will be the ether hypothesis, and with it, the representation of phenomena by partial differential equations, that must be abandoned. The necessity to explain why bodies do not meet any resistance from the ether as they pass through it, and the fact that they do not modify its state, and many other considerations, have created a simple *physical space* out of Fresnel's mechanical ether, perfectly penetrable by matter, *a system of absolute coordinates*. The ether is now only a mathematical abstraction and its elimination would only be the final phase of a long evolution.

This conclusion, as I set it forth, is not at all involved with a return to actions at a distance. Nevertheless, it indeed collides head-on with many currently accepted ideas, and I am the first to admit that a hypothesis which has rendered such great services to Science can't be condemmed for the

sole reason that it presently raises some seemingly inextricable dif-
ficulties. We should always hope for future solutions of these diffi-
culties, or accept the idea that they are inherently a part of things,
and independent of our models. This is, fortunately, not the case.
This is what I have sought to demonstrate in the Second Part of this
work, but the theory which I will present does not pretend to be a
satisfactory and definite solution to a problem so difficult. Its pri-
mary purpose is to show how large is the unknown part which, in spite
of recent advances, still exists in this domain, and in what measure,
[its] much smaller than we would be tempted to believe. Experimental
evidence may be considered as confirmation of Maxwell and Lorentz's
theory, even when we adopt, as I will do, the remarkable ideas of this
latter savant on the atomic constitution of electricity, the nature
of conduction current and of dielectrics, in a nutshell, the theory of 320
electrons. These researches will show that it is not necessary to in-
troduce absolute motion and thus to upset Kinematics and the notion of
time; relative velocities alone will suffice. There will be no use of
notions subject to criticism such as polarization, the electric vector, 321
the magnetic force, etc., but only the notions of time, space, and
electric charges, these latter only playing, like the masses in Mechan-
ics, the role of coefficients, conveniently chosen and invariable for a
given ion or electron. In a certain sense it is a mechanical theory of
electricity. But I have not beleived it advisable to bring in the more
or less complicated latent mechanisms which play such an important rôle
in Maxwell's theory. Those hypotheses are unnecessary, and, one must
say, barely satisfactory. It suffices, indeed, to recall that ponder-
able bodies

must pass through these complicated mechanisms without disturbing them and without feeling sensible action, even when their speed reaches that of celestial bodies. Impenetrability, in particular, doesn't exist in the mechanical [ether] theories, and this is the one point which isn't always sufficiently placed in evidence. Experience has shown that actions are not instantaneous; also it hasn't revealed any trace of a medium which could exist in materially empty space. I therefore felt I could restrict myself to give to the law of propagation of these actions, a very simple kinematic interpretation borrowed form the emanative theory of light and satisfying the principle of relativity of motion. Fictitious particles are constantly emitted in all directions by electric charges; they keep on moving indefinitely in straight lines with constant speed, even through material bodies. The action undergone by a charge depends uniquely on the disposition, velocity, etc., 321
of these particles in its immediate surroundings. The particles are therefore simply a concrete representation of kinematic and geometric data. These hypotheses are sufficient for the purely critical objective that I suggest to reach here. They permit study in detail of the law of elementary actions between electrons in motion and show in particular, that this law, almost entirely unknown for great speeds, requires, even at small speeds, an indeterminant parameter K, which is not without analogy with the one that Helmholtz has introduced in his theory.

I need to specify the temporary scope of these hypotheses. Indeed, when the particles (or, if we like, the actions or energies) emitted by an electrified body reach another electric charge and modify its motion, the principle of action and reaction demands that they undergo, 322
on their part, a deviation, or a change, and it is very remarkable that

[1]

Fizeau's experiment on the entrainment of waves, like certain other facts of Optics, is not compatible with the hypothesis admitted here, and demands such a reaction. It's the opposite that happens in the ether hypothesis, as Poincaré so presented it. Hertz's theory, which satisfies the principle of action and reaction, is incompatible with Fizeau's experiment; Lorentz's theory, which doesn't satisfy it, explains the experiment perfectly. But Poincaré has shown that in giving a momentum to the radiant energy, everything falls into place; therefore this hypothesis is natural if this energy is projected instead of being propagated. It is precisely this that permits safeguarding this principle in the new model that I propose. We can even foresee the possibility of obtaining, by these principles, the electrodynamic terms that 322
depend on velocity and acceleration, using only the consideration of propagation, a problem that Gauss posed in his well know letter to W. Weber, and that Maxwell's theory didn't solve because it introduces to these terms a special quantity, the vector potential.

I will return to these questions later. The remarks which precede are sufficient to explain why I didn't address optics in this criticism.

In many respects, the new theory will therefore bring the reader back to some older classical ideas, which seemed destined to be forgotten. The interpretation of certain experiments will necessarily be modified. In particular, perhaps part of or the whole of mass will be electromagnetic in origin, but it will be constant and won't depend on an absolute velocity. It is the forces, and not the mass, that change. Kaufmann's experiments also permit this new viwpoint.

The new formulae are applicable to gravitation;

they permit abolishment of, at least in great part, the most apparent divergence which exists at this time between calculation and experiment regarding the perihelon motion of Mercury.

The theory of electrons has constituted a first partial return from Maxwell's ideas to others much older, and for those who consider as indispensable a new evolution in the same sense, Lorentz's hypotheses, 323
which have been so fruitful, will maintain their importance, and the mathematical form that he gave them will continue in many cases to be the most elegant and the most practical.

FIRST PART

§ 1. - Recall of Lorentz's Theory[1] 323

We know that Maxwell did not work on hypotheses on the nature of electric current. Lorentz claims that all conduction current results from motion of electric particles, which are subjected to a kind of friction in conductors and to elastic forces in dielectrics. Many experiments have, in the last years, confirmed this hypothesis. This concept has permitted Lorentz to consider only the dielectric ether in his fundamental equations. Renouncing a purely mechanical explanation and the impenetrability of matter, Lorentz considers the ether as immobile and even present in the interior

---

[1] H.A. LORENTZ, Archives neerl., t. XXV, 1892; Versuch einer Theorie der elektrischen und optischen Erscheninungen in bewegten Körpern, Leiden, 1895; Elektronentheorie: Enzyklopädie der math, Wissenschaften, Bd. V, Heft 1, Leipzig, 1904--POINCARÉ, Électricité et Optique, Chap, III, p. 422.

of ions and electrons. The ions and electrons modify it physically, and this modification, which is difficult to picture in a concrete fashion, is characterized by two vectors; the electric or dielectric displacement vector E, whose components are E_x, E_y, E_z, and the magnetic vector H (H_x, H_y, H_z). The electric charges are fixed to the ions which are considered undeformable. Where the electric density, "ρ" is measured in electrostatic units, at x,y,z, at time t, the system of coordinates being connected to the stable ether, and v the velocity of the electric substance in (x,y,z,t) and c the speed of light; we have, for these constraints, the system of equations

$$\text{(I)} \quad \begin{cases} \dfrac{\partial H_z}{\partial y} - \dfrac{\partial H_y}{\partial z} = \dfrac{1}{c}\dfrac{\partial E_x}{\partial t} + 4\pi\rho\dfrac{V_x}{c}, \\[2ex] \dfrac{\partial H_x}{\partial z} - \dfrac{\partial H_z}{\partial x} = \dfrac{1}{c}\dfrac{\partial E_y}{\partial t} + 4\pi\rho\dfrac{V_y}{c}, \\[2ex] \dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = \dfrac{1}{c}\dfrac{\partial E_z}{\partial t} + 4\pi\rho\dfrac{V_z}{c}, \end{cases} \qquad \boxed{\operatorname{curl} H = \frac{1}{c}\frac{\partial E}{\partial t} + 4\pi\rho\frac{V}{c}}$$

323

324

$$\text{(II)} \quad \begin{cases} \dfrac{\partial E_z}{\partial y} - \dfrac{\partial E_y}{\partial z} = -\dfrac{1}{c}\dfrac{\partial H_x}{\partial t}, \\[2ex] \dfrac{\partial E_x}{\partial z} - \dfrac{\partial E_z}{\partial x} = -\dfrac{1}{c}\dfrac{\partial H_y}{\partial t}, \\[2ex] \dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = -\dfrac{1}{c}\dfrac{\partial H_z}{\partial t}, \end{cases} \qquad \boxed{\operatorname{curl} E = -\frac{1}{c}\frac{\partial H}{\partial t}}$$

$$\text{(III)} \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial C_z}{\partial z} = 4\pi\rho, \qquad \boxed{\operatorname{div} E = 4\pi\rho}$$

$$\text{(IV)} \quad \frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0, \qquad \boxed{\operatorname{div} H = 0}$$

$$\text{(V)} \quad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) + \frac{\partial}{\partial z}(\rho V_z) = 0. \qquad \boxed{\frac{\partial \rho}{\partial t} + \operatorname{div} \rho V = 0}$$

The field therefore created in the ether by other charges present, exerts on the element of charge $\rho d\tau$ the mechanical force represented in magnitude and direction by the vector $F\rho d\tau$, where

$$\text{(VI)} \quad \begin{cases} F_x = E_x + \frac{1}{c}\left(v_y H_z - v_z H_y\right), \\ F_y = E_y + \frac{1}{c}\left(v_z H_x - v_x H_z\right), \\ F_z = E_z + \frac{1}{c}\left(v_x H_y - v_y H_x\right). \end{cases} \qquad \boxed{F = E + \frac{1}{c}(v \times H)}$$

In this theory, there is no magnetism[1]; the magnetization comes from currents of particles, according to Ampère.

By means of certain hypotheses, to which we will return, this system of equations integrates itself by the introduction of <u>retarded potentials</u>. We show, as a matter of fact, that all solutions of the system (I) to (V), where we assume the data ρ, v_x, v_y, v_z can be expressed in the form

$$\text{(VII)} \quad \begin{cases} E_x = -\frac{\partial \Phi}{\partial x} - \frac{1}{c}\frac{\partial A_x}{\partial t}, \\ E_y = -\frac{\partial \Phi}{\partial y} - \frac{1}{c}\frac{\partial A_y}{\partial t}, \\ E_z = \ldots, \end{cases} \qquad \boxed{E = -\operatorname{div} \Phi - \frac{1}{c}\frac{dA}{dt}}$$ 324

$$\text{(VIII)} \quad \begin{cases} H_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \\ H_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, \\ H_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}, \end{cases} \qquad \boxed{H = \operatorname{curl} A}$$

the functions Φ (scalar potential) and A_x, A_y, A_z (components of the 325
vector potential) being continuous with their first derivatives in all space, from zero to infinity, and satisfying the equations

$$\text{(IX)} \quad \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} - \Delta \Phi = 4\pi\rho,$$

$$\text{(X)} \quad \begin{cases} \frac{1}{c^2}\frac{\partial^2 A_x}{\partial t^2} - \Delta A_x = \frac{4\pi\rho v_x}{c}, \\ \frac{1}{c^2}\frac{\partial^2 A_y}{\partial t^2} - \Delta A_y = \frac{4\pi\rho v_y}{c}, \\ \ldots\ldots\ldots\ldots\ldots\ldots \end{cases}$$

[1] Magnetic "poles" as distinct independent entities are not considered.

and

(XI) $$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = -\frac{1}{c}\frac{\partial \phi}{\partial t}.$$ $\boxed{\operatorname{div} A = -\frac{1}{c}\frac{\partial \phi}{\partial t}}$

Lorentz satisfies these conditions by setting

(XII) $$\begin{cases} \phi(x,y,z,t) = \iiint \frac{[\rho']}{r}\, d\tau', \\ r^2 = (x-x')^2 + (y-y')^2 + (z-z')^2, \end{cases}$$

(XIII) $$\begin{cases} A_x = \frac{1}{c}\iiint \frac{[\rho' V_x']}{r}\, d\tau', \\ A_y = \frac{1}{c}\iiint \frac{[\rho' V_y']}{r}\, d\tau', \\ A_z = \ldots\ldots \end{cases}$$

These expressions have the form of newtonian potentials, with the difference that instead of taking the value of ρ at point $x'y'z'$ at time t, we have to take it at the previous time $t'=t-r/c$, the time r/c being necessary because of the propagation; this is what we will point out along with Lorentz by the notation 325

$$[\rho'] = \rho\left(x', y', z', t-\frac{r}{c}\right),$$

$$[\rho' V_x'] = \rho\left(x', y', z', t-\frac{r}{c}\right) V_x\left(x', y', z', t-\frac{r}{c}\right).$$

The field is therefore completely determined, and in introducing the expressions (XII) and (XIII) into ~~in the~~ formulae (VII), (VIII) and [hence into] (VI), we obtain for F_x 326

(XIV) $$F_x = \iiint d\tau' \Big\{ -\frac{\partial}{\partial x}\frac{[\rho']}{r} - \frac{1}{c^2}\frac{\partial}{\partial t}\frac{[\rho' V_x']}{r} + \frac{V_x}{c^2}\frac{\partial}{\partial x}\frac{[\rho' V_x']}{r} + \frac{V_y}{c^2}\frac{\partial}{\partial x}\frac{[\rho' V_y']}{r} + \frac{V_z}{c^2}\frac{\partial}{\partial x}\frac{[\rho' V_z']}{r} - \frac{V_x}{c^2}\frac{\partial}{\partial x}\frac{[\rho' V_x']}{r} - \frac{V_y}{c^2}\frac{\partial}{\partial y}\frac{[\rho' V_x']}{r} - \frac{V_z}{c^2}\frac{\partial}{\partial z}\frac{[\rho' V_x']}{r} \Big\},$$

and similar expressions for F_y, F_z. In introducing the total derivative

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z},$$

and setting

$$\text{(XV)} \quad L(x, y, z, t, v_x, v_y, v_z) = \iiint \frac{[\rho']}{r} \left\{ 1 - \frac{v_x[v_x'] + v_y[v_y'] + v_z[v_z']}{c^2} \right\} d\tau'$$

Schwarzschild[1] has found for F_x the remarkable form

$$\text{(XVI)} \quad F_x = -\frac{\partial L}{\partial x} + \frac{d}{dt}\frac{\partial L}{\partial v_x}, \quad F_y = \ldots .$$

This is the form of Lagrange's equations. The expressions (XIV) and (XVI) give the force undergone by an electric point of unit charge, expressed by means of elementary actions analogous to the ones we considered in the old Electrodynamics, without the notion of non instantaneous transmission, that we find again in Gauss and C. Neumann. A charge e', sensibly point-like, exercises, under very general conditions, on another similar charge e, a force 326

$$\text{(XVII)} \quad \begin{cases} ee'\left(-\frac{\partial \mathcal{L}}{\partial x} + \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial v_x}\right), \\ ee'\left(-\frac{\partial \mathcal{L}}{\partial y} + \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial v_y}\right), \\ \ldots\ldots\ldots\ldots\ldots \end{cases}$$

where 327

$$\mathcal{L} = \frac{1}{r\left\{1 - \frac{[v_r]}{c}\right\}} \left\{ 1 - \frac{v_x[v_x'] + v_y[v_y'] + v_z[v_z']}{c^2} \right\},$$

$$r^2 = \left[x(t) - x'\left(t - \frac{r}{c}\right)\right]^2 + \left[y(t) - y'\left(t - \frac{r}{c}\right)\right]^2 + \left[z(t) - z'\left(t - \frac{r}{c}\right)\right]^2.$$

---

[1] Gottinger Nachr., Math.-Phys. Klasse, 1903, p. 126.

This expression reduces itself, to a first approximation, to the law of distance squared. We can therefore name it the generalized Newton's law. This explicit expression will be given later on.

In these formulae, the notion of field doesn't come into play. It is very remarkable that Clausius, like Weber, in seeking to account for the electrodynamic actions, by means of actions at a distance, depending on positions, velocities and accelerations of the electric points, has derived the same formulae (XV) and (XVI), with the only difference that the actions are instantaneous, so that we have to take the values of ρ' and ν' at time t, and not at time $t-\frac{r}{c}$.

This very remarkable result, due to Schwarzschild, shows that Lorentz's theory is very close to the old theories.

The first form that Lorentz gave to his theory was less abstract, 327
in the sense that, following the path that Maxwell laid out, he started from Lagrange's equations, introducing two kinds of variables, the first ones determining the positions of the electrified particles, the others the state of the ether. We attribute to this latter one [the ether] kinetic energy, without determining its internal motions. It suffices that they exist. Hamilton's principle permits, likewise, restricting the variations under certain conditions, to obtain the fundamental equations (I) to (VI) by considering the electrical energy

$$\frac{1}{8\pi}\int\left(E_x^2+E_y^2+E_z^2\right)d\tau$$

as potential energy, and the magnetic energy

$$\frac{1}{8\pi}\int\left(H_x^2+H_y^2+H_z^2\right)d\tau$$

as kinetic energy[1]. This deduction is fairly complicated and can be done in various forms[2]. 328

These two aspects of Lorentz's theory are clearly distinctive. The second one is similar to Larmor's theory[3], which, in leading to the same formulae, offers precise hypotheses on the motions of ether in an electromagnetic field, as borrowed from Lord Kelvin's conceptions on gyrostatic ether. Ether is incompressible and moves itself in the direction of magnetic force lines.

We know that Maxwell and Hertz explain the mechanical forces that a substance experiences in an electromagnetic field by the pressure that ether is supposed to exert on the substance, and conversely: the action of one is equal and opposed to the reaction of the other at all points. These pressures, as Helmholtz has shown[4], tend to set the 328
ether (supposedly incompressible) into motion; they are, by unit volume, proportional to the time derivative of Poynting's radiant vector S:

$$\text{(XVIII)} \quad \begin{cases} S_x = \frac{1}{4\pi c}\left(E_y H_z - E_z H_y\right), \\ S_y = \frac{1}{4\pi c}\left(E_z H_x - E_x H_z\right), \\ S_z = \frac{1}{4\pi c}\left(E_x H_y - E_y H_z\right). \end{cases} \qquad \boxed{S = \frac{1}{4\pi c} E \times H}$$

Lorentz considers ether as immobile; he is therefore led to abandon Maxwell's theory of pressures

---

[1] LORENTZ, Proc. Amsterdam Acad., 1903, p. 608; Elektronen-theorie, p. 165 and 170.

[2] LARMOR, Aether and Matter, Cambridge, 1903, chap. IV. - SCHWARZSCHILD, Gottinger Nachr., 1903, p. 125.

[3] LARMOR, Proc. Roy. Soc., t. LIV, 1893, p. 438; Aether and Matter.

[4] Gesammelte Abhandlungen, t. III, p. 526.

and, by that, the equality of action and reaction, the non-compensated force being characterized by the vector $\frac{\partial S}{\partial t}$.[1]

Lets observe, in concluding this brief exposition, that if we admit unknown motions in ether, the solution of the equations is determinable only for quantities close to the order of the speed of ether divided by that of light.

329

§ 2. -- CRITICSM OF NOTIONS OF ELECTRIC AND MAGNETIC FIELDS

We know that the introduction of the notion of force into mechanics was the subject of much criticism. This notion calls for muscular sense, whereas the ideas of space and time are primarily of tactile and visual origin: and the irreducible psychological 329
duality introduced at the very base of this fundamental science leaves a certain discomfort in the mind, justifiably so, for it seems very obvious that the notion eliminates itself in each particular case. Whether we measure the forces by masses and accelerations, or by elastic deformations, whether we oppose their effects with those of gravity, etc., what we really observe and measure is always a displacement, or the absense of a displacement; again, in this latter case, we only end up defining the difference of two forces. In the equations of mechanics, as applied to any particular example, there remain only the relations of space and time, with certain coefficients properly chosen and invariable which are the masses or other physical constants. With

---

[1] POINCARÉ, <u>Electricite et Optique</u>, p. 448.

regard to pure logic, it is therefore with good reason that many scholars have rejected the introduction of the notion of force in the fundamental expressions as being useless.

Modern Electrodynamics is entirely based on the notions of electric and magnetic forces. If this was absolutely necessary it would be regrettable. But it isn't so: these notions eliminate themselves in the equations, they are logically useless; the theory only expresses (in the last analysis) the existence of certain relations of time and space, as it is in the case of Mechanics. It will therefore be preferable to express these relations directly: we thus come back to the classical elementary actions.

In fact, what are the exact definitions of the vectors for E and H fields? I say that these vectors are defined by the theory itself. Thus, without knowing the significance of these symbols, we can at once, by means of certain hypotheses that we will examine in the next section, integrate the fund- 330
amental equations by the method of retarded potentials, and we will be lead to expressions (XIV) or (XVI). The equations of motion for a material point of charge e, of mass m and of coordinates x, y, z, will be

$$(1)\qquad \begin{cases} m\dfrac{d^2x_1}{dt^2} = eF_x\left(x_1, y_1, z_1;\ V_{x_1}, V_{y_1}, V_{z_1}, t\right), \\ m\dfrac{d^2y_1}{dt^2} = \ldots\ldots \end{cases}$$

If we desire to take into account the action of the electron on itself, or of liaison, d'Alembert's principle has to be applied, and we have, in extending the integration over the whole of the electron, and designating by $\delta x_1, \delta y_1, \delta z_1$ virtual displacements compatible with the liaisons, by μ_1, ρ_1

the densities of the substance and electricity,

$$(2) \quad \int \Big\{ \Big[\mu_1 \frac{d^2x_1}{dt^2} - \rho_1 F_x(x_1, \ldots)\Big] \delta x_1 + \Big[\mu_1 \frac{d^2y_1}{dt^2} - \rho_1 F_y\Big] \delta y_1 + \Big[\mu_1 \frac{d^2z_1}{dt^2} - \rho_1 F_z\Big] \delta z \Big\} = 0.$$

After having replaced F_x, . . with the value (XIV) or (XVI) (the terms relative solely to the electron will play a special role), we will have, in (1) and (2), only relations of space and time, even when $\mu = 0$, that is to say when the mass is entirely of electromagnetic origin.

Now, I say that Lorentz's equations don't effectively express anything more than (1) and (2). That is to say, that the field never plays a role in pure ether. In fact, we can only determine the field's magnitude and direction by placing a body and observing the mechanical forces that it feels or rather its motions and those of the ions in
its near vicinity, motions which are indicated by luminous, thermal, 330
chemical, etc, phenomena. Therefore, we only know F, and, that, only in points of x_1, y_1, z_1, where there is electrified matter, and we deduce E and H by reasoning (which is not always so simple when we have to consider absolute motion). This is to say that it will suffice,
in all cases, to know the formula that gives F as the result of ele- 331
mentary actions exerted by an element of charge on another element of charge, and that this second representation is, with regard to the facts, exactly equivalent to the first one, which ~~is based on the field~~ and its partial differential equations which only play a purely mathematical role. We can, if we please, dispense

Translated (1980) from Ann. de Chim. et Phys., 8^{e} serie, t. XIII (Fevrier 1908) completely with the notions of electric and magnetic fields.

It is important to specify the sense of this affirmation. In the theory of light, for example, everything thus presented with regard to Lorentz's theory can be derived from elementary actions between ions in the luminous source, the ones in the dielectrics or conductors which constitute the optical apparatus, and finally the ones in the retina or sensitive plate which receives the impression. Thus, we are accustomed, for example, to describe the phenomena of diffraction, that we observe in the case of a slit used with a screen, by considering with Fresnel that the points of ether situated in the slit as so many centers of disturbance. This does not conform to the formula for retarded potentials: electric charges are the only points of origin for waves. Lorentz's theory, or the law of elementary actions, will explain them as the combined action of ions in the source and in the screen; besides, 331
it is easy to show, using Huygens' principle in the form that Kirchoff has given it, the equivalence of the two procedures from the viewpoint of results.

It wouldn't be permissible any more to say that the field is a purely mathematical intermediate with which we can dispense, *if it were possible to perceive its existence in a point of ether without placing any matter at the point*. This could happen, for example, if ether were, under the influence of a field, susceptable to modification or to move more or less as Hertz's theory wants it to, and as Lord Kelvin demands[1]. Interference experiments could have placed this speed into evidence. These ideas were generally very widespread;

---

[1] LORD KELVIN, *Baltimore Lectures on molecular dynamics and the wave theory of light*, London, 1904, p. 159: "It is absolutely certain that there is a definite dynamical theory for waves of light, to be enriched, not abolished, by electromagnetic theory."

but we know that the experiment, examined carefully repeatedly,[1] gave only negative results, as do all the experiments designed to prove the existence of ether. The hypothesis of the motions, on the other hand, has not lent itself to any plausible mechanical explanation of electrical actions in their effects. Lorentz was therefore lead to exclude it in the most recent expositions of his theory; and this is what leads to the affirmation that we can eliminate the notions of force and field in this theory without affecting any actual or possible fact of experience according to it.

Lorentz had already indicated[2] this point of view; "We therefore see, in the new way I'm going to present it, Maxwell's theory draws nearer to the older ideas. We can even, after we have established the simple formulae that describe the motions of particles, leave out the reasoning that spawned them and consider looking at these formulae as expressing a fundamental law comparable with those of Weber and Clau- 332
sius." The actions, however, are not instantaneous anymore; and we have seen, that except for this important restriction, there is even an identity with Clausius' law.

We can easily see that the notion of field introduces that of absolute motion, as soon as the velocities come into play, either in the field expressions or in that for its action on bodies. It isn't likely anymore that it depends only on coordinates and accelerations.

§ – 3. Irreversibility and Retarded Potentials

Now I intend to examine more closely the hypothesis which relates differential equations

---

[1] See in particular O. LODGE, Phil. Trans., t. CLXXXIV, 1893; HENDERSON and HENRY, Phil. Mag., 5e serie, t. XLIV, 1897, p. 20.

[2] Arch. neerl., t. XXV, 1892, p. 433.

(IX) and (X) to ~~the~~ retarded potentials formulae (XII) and (XIII) and to show that the transformation of the latter ones to the first ones is immediate, but that the inverse proposition is far from being true.

First we have to recognize the fundamental importance of the formulae. In contrast to mechanical phenomena, electro- 333
magnetic phenomena are, in general, irreversible because of radiation; we can, by this motive, even hope to get by their means a more complete interpretation of irreversible physical phenomena. But Lorentz's equations don't change when we change the direction of time; they contain the affirmation of reversibility, whereas for retarded potentials and elementary actions, the positive and negative directions of time play entirely
different roles. We insert, as in Helmholtz's irreversible cycles, 333
a ~~speed~~ which is hypothetically impossible to change the sense: the ~~speed~~ with which waves constantly move away from the bodies that generated them. It is from this, that electromagnetic irreversibility is derived. This additional hypothesis which preceeds must therefore be examined with care.

With $f(x,y,z,t)$ a continuous function, proportional to the electric density $\rho(x,y,z,t)$ and φ another function having the property of continuity of potential in all space and at infinity and satisfying, everywhere, the equation

$$(3) \qquad \frac{1}{c^2}\frac{\partial^2\varphi}{dt^2} - \Delta\varphi = 4\pi f$$

it is easy to verify that the integral

$$(4) \qquad \begin{cases} \varphi_1(x,y,z,t) = \int d\tau' f\left(x',y',z',t'-\frac{r}{c}\right) \\ \left[r^2 = (x-x')^2 + (y-y')^2 + (z-z')^2\right] \end{cases}$$

is a solution of (3). In fact, lets isolate a small volume τ_0 around point xyz; we'll be able to differentiate the integral under the summation sign relative to the rest of space for which xyz is an external point; in applying the operation $\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta$, we find zero. Inside τ_0 the $\frac{\partial^2}{\partial t^2}$ operation is performed again, under the summation sign, and gives a result closing to zero with τ_0. To do the Δ operation lets put r, being very small, into τ_0,

$$\int_{\tau_0} \frac{d\tau' f\left(x', y', z', t - \frac{r}{c}\right)}{r} = \int_{\tau_0} d\tau' \left[\frac{f(x', y', z', t)}{r} - \frac{1}{c}\left(\frac{\partial f'}{\partial t}\right)_{t + \frac{\vartheta r}{c}} \right].$$ 334

The $-\Delta$ operation applied to the first term gives $4\pi f(x,y,z,t)$ according to Poisson's theorem. In the second term the denominator r is missing. The result closes to zero with τ_0; we therefore obtain equation (3). I don't insist on the condition of continuity lightly, in that we have to insure that there are 334
derivatives of f.

We demonstrate that

$$\varphi_2 = \int d\tau' \frac{f\left(x', y', z', t + \frac{r}{c}\right)}{r},$$

$$\varphi_3 = \frac{1}{2}\int \frac{d\tau' \left[f\left(x', y', z', t - \frac{r}{c}\right) + f\left(x', y', z', t + \frac{r}{c}\right)\right]}{r}$$

and, more generally,

$$\varphi_4 = \int \frac{d\tau'}{r} \left[F_1\left(x', y', z', t - \frac{r}{c}\right) + F_2\left(x', y', z', t + \frac{r}{c}\right)\right]$$

are again solutions of (3), provided that the arbitrary functions F_1 and F_2 satisfy the relation

$$F_1(x, y, z, t) + F_2(x, y, z, t) = f(x, y, z, t).$$

The solution φ_1 corresponds to waves which move away in all directions from the electric charges that generated them. φ_2 is for waves coming from infinity converging on these same points. In contrast to φ_1, which depends only on previous states, φ_2 depends on subsequent states. The solution φ_3 contains both kinds of waves. And finally, φ_4 corresponds to waves whose centers of disturbance may be located in pure ether, where *f* = 0. Experience shows, and Lorentz admits, that only φ_1 waves can exist, and besides, we will see that contrary hypotheses would involve inadmissible consequences, such as the possibility of *perpetual motion*. We conclude, to begin with, that *Lorentz's equations* (and the **result extends to those of** Maxwell and Hertz) *accept an infinite number of solutions,*
satisfying all conditions, but incompatible with experience. 334

We readily find such solutions, and are lead to set them
aside *a priori*, each time we calculate, for example, the 335
electrical oscillations of a system (a conducting sphere, Hertz's excitor, an oscillating electron, etc.).

Lets discuss the hypothesis by which we believe we can eliminate these solutions. We show that we have for all solutions of (3) inside a closed surface S

$$(5) \quad \varphi(x,y,z,t) = \int \frac{[f']}{r}\,d\tau' + \int_{S'} dS' \left\{ [\varphi'] \frac{\partial \frac{1}{r}}{\partial n} - \frac{1}{r}\left[\frac{\partial \varphi'}{\partial n}\right] - \frac{1}{cr}\frac{\partial r}{\partial n}\left[\frac{\partial \varphi'}{\partial t}\right] \right\},$$

where we have set, as before,

$$[f'] = f\left(x', y' z', t - \frac{r}{c}\right).$$

Suppose that at any instant *t* = 0 *situated*

<u>in the past</u> we had everywhere, or at least at great distances,
$\varphi = 0$, $\frac{\partial \varphi}{\partial t} = 0$; we could, for all positive values of t and at all
points xyz, choose for S a sphere of center xyz and radius $R > ct$
big enough so that all terms of the surface integral be zero, formula (4) will remain. This reasoning calls for the following
remarks:

1° The terms for uniform translational or rotational motion
which enter into the electromagnetic theory never satisfy the
relative conditions at $t = 0$; this theory therefore remains excluded.
More generally, the authors who have used this reasoning, once the
formula was set up, didn't bother to verify, in the equations they
dealt with, if this condition is fulfilled. It isn't in most
cases. Now we have seen that the formula must be <u>absolutely general</u>.

2° If for $t = 0$ there is only a very weak field at great distances,
this field, if it is for a convergent wave, could acquire, a few 335
moments later, a great intensity at a given point in space. It is
not sufficient therefore to suppose a weak field for the moment
$t = 0$ for all space (or at least at great distances); it has to be
rigorously zero (which is an hypothesis of admissible character in
physics) or that had been previously excluded in the convergent
waves, which would be a petition of principle. In the case of
sound (and this analogy can mislead easily) friction destroys the 336
wave entirely after a few moments, and the reasoning is practically
the same; it is not the same though for ether, and we should expect,
<u>a priori</u>, to find a similar state of affairs to the one we observe
at seaside, where in addition to the divergent waves produced by
solid bodies of the shore, there are others

which constantly come in from the sea and are not produced by solid bodies. So if formula (4) is not rigorous, we would have to expect at each moment the sudden creation of an intense field, a kind of electromagnetic wave coming from infinity or which diverges from a region of ether through which it just converged.

3° Only solar and stellar radiation, which has been creating an oscillating electromagnetic field throughout the universe daily for an extremely long time, could oblige moving the instant $\underline{t} = 0$ back beyond all limits of cognizance. A hypothesis so fundamental must not present this inadmissible character.

4° Lets examine what must have happened before the moment $\underline{t} = 0$. We will find, in changing $\underline{c}$ into $-\underline{c}$, the analogous formula

$$(6)\left\{\begin{aligned} &\phi(x,y,z,t) \\ &\quad = \int \frac{(f')}{r}\,d\tau' + \int ds' \left[(\phi') \frac{\partial \frac{1}{r}}{\partial r} - \frac{1}{r}\left(\frac{\partial \phi'}{\partial n}\right) + \frac{1}{c}\left(\frac{\partial \phi'}{\partial t}\right)\frac{1}{r}\frac{\partial r}{\partial n}\right] \\ &\qquad\qquad \left[(f') = f\left(x', y', z', t + \frac{r}{c}\right)\right] \end{aligned}\right.$$ 336

The same reasoning will give

$$\phi = \int \frac{(f')}{r}\,d\tau'$$

that is to say that before the instant $\underline{t} = 0$ the waves were convergent, the bodies became excited by the radiation and thus produced <u>perpetual motion</u>.

It is useless to dwell any longer. <u>The hypothesis that we start from rest (or the unimportant modifications that were presented) is not admissible as a basis for the general law for retarded potentials</u>. It isn't even for the particular cases. Lets consider an Hertzian oscillator: at instant $\underline{t} = 0$ a spark jumps, the magnetic field, at first nil everywhere, is disturbed;

but, after a short time, the system is back at rest. It is not exactly at rest, any more than it was before the experiment. It is 337
in that state only sensibly. If we begin our reasoning during the first state of rest, there would only be converging waves. Why do we choose the first state of rest but have no inclination for doing a new experiment wherein the second state of rest plays the same role as the first? It is because the elements away from the immediate area, which are inaccessible to the experiment, play a preponderant role in the hypothesis. If they were to send us convergent waves, our approximate reasoning, based on the close-in elements, would soon cease to give an approximation, even a coarse one.

Fortunately, we know, apriori, by means of a lengthy experiment, that the distant waves diverge. This is what allows us to ignore them and also renders the demonstration unnecessary. If ether had vis-
cosity, similar to that of air, very different considerations would 337
ensue and there wouldn't be any reason to be surprised by irreversibility, since we would have introduced it in the equations themselves.

More generally, any theory which makes more specialized hypotheses about the initial state, than the law of potentials requires, will not be admissible. It would exclude some actual phenomena and would allow, for $t<0$, impossible solutions. Therefore, what are the initial conditions, necessary and sufficient, for (4) to hold?

Lets postulate

$$\phi = \phi_1 + \psi$$

We will have

$$\frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} - \Delta \psi = 0 \tag{7}$$

in all space; $\psi=0$ ~~at~~ t_0 infinity. For ψ to be constantly uil, it necessary and sufficient that for $t=0$ we have

$\psi = 0, \frac{\partial\psi}{\partial t} = 0$ in all space. This says that for **formula** (4) to be applicable over all time, it is necessary and sufficient that it hold for two instants, t = 0, and t = dt infinitely close, a statement about which there is evidently nothing to be drawn from Maxwell's ~~viewpoint.~~ point of view.

Can we replace the initial state hypothesis by some other equivalent general condition? 338

Lorentz[1] doesn't make use of the hypothesis. He simply admits that the surface integral in (5) is nil when S moves away indefinitely. After having written (4), he continues "This solution is not therefore the general integral of (3); there will also be, for example, solutions corresponding to a movement of waves which would be directed towards the element of volume instead of moving away from it. We will reject them from the theory by admitting, once and for all, that charged elements are the only points of origin for disturbances. We exclude also 338 the states of ether which are completely independent of the charged substance: which if non-existent, the ether would remain continually at rest."

But lets apply, these ideas, without change, to **formula** (6). As the surface integral vanishes we would get convergent waves. But neither method of proceeding is admissible. These surface integrals, considered as functions of x,y,z,t, are general solutions of (6). They would not therefore approach zero, for a given value of x,y,z,t, when the surface moves away indefinitely: they have an invariable value, finite or nil, according to whether the chosen

---

[1] Elektronetheorie, p 158; see also WIECHERT, Arch. neerl., 1900, p 549, and P. HERTZ, Untersuchungen uber unstetige Bervegunen eins Elektrons Inaug. Diss., Gottingen, 1904, p5 and 12, and note. In his first thesis of 1892, Lorentz contented himself with verifying that satisfactory equations exist.

solution is finite or nil. There is nothing to draw from such identities. Finally, there is no precise sense to attach to this proposition: the disturbances that are dependent on ether are the only ones to be excluded. If we could use, a priori, formula Φ_2 or Φ_3, which depend only on the material, we could write

$$\Phi_1 = \Phi_2 + \bar{\Phi}, \text{ or } \frac{1}{c^2}\frac{\partial^2 \bar{\Phi}}{\partial t^2} - \Delta \bar{\Phi} = 0,$$

and we would still have the right to say that $\bar{\Phi}$ is independent of the material, since its differential equation and its conditions of continuity are also independent. In this manner of examination, it is Φ_1 which would contain a term that is independent of the material. Finally, will we say that the state at a given point is determined by the "previous history" of the material only? Solutions other than (3) are still possible.

The ~~inadequacy~~ insufficiency of these statements, and others analogous, is 339
that they hold that, the decomposition of a field of waves is a mathematical operation which can be done in an infinite number of ways. But the character of this operation is doubly artificial from the viewpoint of Maxwell's ideas, because consideration of the origin of the waves demands consideration of the entire field during a finite interval of time, whereas Maxwell saw an essential advantage in his theory, precisely from the fact, that it makes it unnecessary to consider elementary actions and the field's origin and only concerns itself with the immediate surroundings of the point under consideration. We see that these statements are useless, and that to eliminate the physically impossible solutions, it only requires adoption a priori of the formulae for retarded potentials, which distinguish the elementary actions like the classical theories did, and to prove that they satisfy the equations, is to say that they can replace them completely, whereas the inverse is not the case.

In bringing these results closer to the ones in the preceeding paragraph, we see that in the last analysis it is the formula of elementary actions, and not the system of partial differential equations, which is the complete and exact expression of Lorentz's theory.

We have to add the hypothesis of absolute coordinates. We have just seen that ether, instead of playing an independent role, and even preponderant, as we should have expected since it is supposed to be the reservoir of all electromagnetic energy, steals away once more. Its only role will consist in providing, though contradicting experience, a system of absolute coordinates.

We will evidently have the same difficulties in passing from any kind of system of partial differential equations, reversible
at least wherein it concerns pure ether, to irreversible solutions 339
that the experiment demands. In Hertz's theory, even this seems impossible. The partial differential equations and the notion of ether are essentially inappropriate to express the comprehensive laws for the propagation of electrodynamic actions.

340

§ 4.- ENERGY

Maxwell has shown, in all the particular cases with which we are acquainted regarding electromagnetic energy, that the energy can be expressed in the form of an integral extended over all space, which in Lorentz's hypothesis takes the form

$$W = \frac{1}{8\pi}\int\left(E_x^2+E_y^2+E_z^2+H_x^2+H_y^2+H_z^2\right)d\tau.$$

Maxwell admits, and this is an important point in his system, that each element of volume is effectively, and in all cases, the seat of a quantity of energy equal to

$$\frac{\left(E^2+H^2\right)}{8\pi}\,d\tau.$$

Poynting has since shown that this results from the general equations, that for all closed surfaces σ we have

$$dT + dW = -dt \int S_n \, d\sigma,$$

where dT is the work of outside forces, and S_n is the surface component in the direction of the normal external radiant vector

$$S_x = \frac{c}{4\pi}\left(E_y H_z - E_z H_y\right), \quad S_y = \frac{c}{4\pi}\left(E_z H_x - E_x H_z\right), \quad S_z = \ldots \qquad \boxed{S = \frac{c}{4\pi} E \times H}$$

The theorem states itself elegantly in considering the energy as being comprised of an indestructable fluid which moves parallel to the radiant vector, an image which presents certain advantages but lays itself open to much criticism.[1] Particularly, we can ask ourselves if the statement "continuous energy in a given volume with such and such value," has any real sense when we can only define differences of energy. 340

We will set aside this metaphysical question, to study the indetermination that this mode of representation suffers. The electro- 341 static energy of a system is

$$(8) \quad \left\{ \begin{array}{l} W_E = \dfrac{1}{2} \displaystyle\iint \frac{\rho_1 \rho_2 \, d\tau_1 \, d\tau_2}{r_{12}} \\ \left[\begin{array}{l} \rho_1 = \rho(x_1, y_1, z_1), \quad \rho_2 = \rho(x_2, y_2, z_2) \\ r_{12}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \end{array} \right] \end{array} \right. ,$$

the two integrations being extended over all space wherever electric charges are found.

It is evident that this expression can be put in the form of an extended integral throughout all space in an infinite number of ways and the one Maxwell has chosen is, in the point of view of his system, particularly simple. But we can show others which will have, for example,

1 See W. WIEN, Annalen der Physik u. Chemie 2e serie t. XLV, 1892, p 684

the advantage of drawing closer to the forms used in the theory of elastic bodies. Lets therefore introduce, instead of the electric force E in x,y,z which is

$$E_x = \int \rho_1 \frac{x-x_1}{r_1^3}\, d\tau_1, \quad E_y = \ldots,$$

the vector

$$(9) \qquad \xi = \int \rho_1 \frac{(x-x_1)}{r_1^2}\, d\tau_1, \quad \eta = \int \rho_1 \frac{(y-y_1)}{r_1^2}\, d\tau_1, \quad \zeta = \ldots,$$

for which

$$(10) \quad \begin{cases} \dfrac{\partial \xi}{\partial x} + \dfrac{\partial \eta}{\partial y} + \dfrac{\partial \zeta}{\partial z} = \displaystyle\int \frac{\rho_1}{r_1^2}\, d\tau_1, \\ \dfrac{\partial \zeta}{\partial y} - \dfrac{\partial \eta}{\partial z} = 0, \quad \dfrac{\partial \xi}{\partial z} - \dfrac{\partial \zeta}{\partial x} = 0, \quad \dfrac{\partial \eta}{\partial x} - \dfrac{\partial \xi}{\partial y} = 0. \end{cases}$$

Now, if ξ, η, ζ were the components of displacement for an elastic body, the energy of this latter vector would be, as we know

$$(11) \quad \begin{cases} W' = \displaystyle\int \Big\{ \lambda \Big(\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} + \frac{\partial \zeta}{\partial z} \Big)^2 \\ \qquad + \mu \Big[\dfrac{\partial \xi}{\partial x}\dfrac{\partial \eta}{\partial y} + \dfrac{\partial \eta}{\partial y}\dfrac{\partial \zeta}{\partial z} + \dfrac{\partial \zeta}{\partial z}\dfrac{\partial \xi}{\partial x} - \dfrac{1}{4}\Big(\dfrac{\partial \xi}{\partial y} + \dfrac{\partial \eta}{\partial x} \Big)^2 \\ \qquad - \dfrac{1}{4}\Big(\dfrac{\partial \eta}{\partial z} + \dfrac{\partial \zeta}{\partial y} \Big)^2 - \dfrac{1}{4}\Big(\dfrac{\partial \zeta}{\partial x} + \dfrac{\partial \xi}{\partial z} \Big)^2 \Big] d\tau, \end{cases}$$ 341

λ and μ being constants of elasticity of the body, or, in adding the zero terms $\frac{1}{4}\left(\frac{\partial \zeta}{\partial y} - \frac{\partial \eta}{\partial z}\right)^2$, . . . and transforming 342

$$W' = \lambda \int \Big(\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} + \frac{\partial \zeta}{\partial z} \Big)^2 d\tau$$
$$\qquad - \mu \int_S \Big[\eta \Big(\frac{\partial \xi}{\partial x} \cos ny - \frac{\partial \xi}{\partial y} \cos nx \Big) + \ldots \Big] dS'$$

If the elastic body is infinitely extended, by virtue of formulas (9), the surface integral will vanish, and leave

$$W' = \lambda \int_\infty \Big(\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} + \frac{\partial \zeta}{\partial z} \Big)^2 d\tau = \int_\infty \Big(\int \frac{\rho_1 d\tau_1}{r_1^2} \Big) d\tau.$$

Electricity, supposedly being firmly attached to small solid bodies (ions) $\underline{k_i}$, we have

$$\left(\int\frac{\rho_1\,d\tau_1}{r_1^2}\right)^2 = \left(\sum\int_{k_i}\frac{\rho_1\,d\tau_1}{r_1^2}\right)^2$$

$$= \sum\left(\int_{k_i}\frac{\rho_1\,d\tau_1}{r_1^2}\right)^2 + \sum_{m,n}\sum\int_{k_m}\int_{k_n}\frac{\rho_1\rho_2\,d\tau_1\,d\tau_2}{r_1^2\,r_2^2}\,.$$

The first term doesn't depend on the mutual positions of bodies; its integral with regard to $\underline{x,y,z}$, extended over all space, will be a constant. In the double summation, the variables $\underline{x_1,y_1,z_1,x_2,y_2,z_2}$ assume the gamut of relative values for a given combination of different ions $\underline{k_m},\underline{k_n}$, we therefore never have $\underline{x_1=x_2}$, $\underline{y_1=y_2}$, $\underline{z_1=z_2}$ which allows the transposition of the order of integration and we write

$$W' = \lambda\sum_m\sum_n\int_{k_m}\int_{k_n}\rho_1\rho_2\,d\tau_1\,d\tau_2\int_\infty\frac{d\tau}{r_1^2 r_2^2}\,.$$

To evaluate the integral taken with respect to $\underline{x,y,z}$ let's introduce polar coordinates r, ϑ, φ, with $\underline{x_1,y_1,z_1}$ as the polar origin, the line 342
joining $(\underline{x_1,y_1,z_1})$ and (x_2,y_2,z_2) as the polar axis. Let $\underline{r_{12}}$ be the distance between these two points; we have to calculate

$$\int\frac{d\tau}{r_1^2 r_2^2} = \int_0^{2\pi}d\varphi\int_0^{\pi}d\vartheta\int_0^\infty\frac{\sin\vartheta\,dr}{r_{12}^2+r^2-2r_{12}\,r\cos\vartheta}$$

$$= 2\pi\int_0^\pi\frac{d\vartheta}{r_{12}}\left(\operatorname{arc\,tang}\frac{r-r_{12}\cos\vartheta}{r_{12}\sin\vartheta}\right)_{r=0}^{r=\infty}\,.$$ 343

The principal values (included between $-\frac{\pi}{2}$ and $+\frac{\pi}{2}$) are $\frac{\pi}{2}$ for $\underline{r} = \infty$, and $\vartheta-\frac{\pi}{2}$ for $\underline{r} = 0$; finally we find

$$2\pi\int_0^\pi\frac{d\vartheta(\pi-\vartheta)}{r_{12}} = \frac{\pi^3}{r_{12}}\,,$$

from which

$$(11)\qquad W' = \sum_{m,n} \pi^3\lambda \int_{k_m}\int_{k_n} \frac{\rho_1\rho_2\, d\tau_1\, d\tau_2}{r_{12}} + \text{const.}$$

We conclude finally, in choosing $\lambda = \frac{1}{2\pi^3}$, that with the hypotheses made and with the units introduced, the elastic energy W' will, having a nearly constant summation, be equal to the electrical energy W_E given by (8), and that we have in particular the formula

$$(12)\qquad W_E = \frac{1}{2\pi^3}\int\left(\frac{\partial\xi}{\partial x} + \frac{\partial\eta}{\partial y} + \frac{\partial\zeta}{\partial z}\right)^2 d\tau$$

which is irreducible to Maxwell's formula, and gives a completely different distribution for the energy. It would be easy to obtain, in starting in this manner, a similar expression for magnetic energy and, consequently, for currents; and we see that, even in insisting on the simplest of formulae, the localization of energy is still an 343
indeterminant problem.

It is the same for the flux of energy.[1] We can always modify the motion of the flowing energy in an arbitrary way by adding to Poynting's vector another vector (u,v,w) obliged only to satisfy the equation for incompressible fluids.

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0 \qquad \boxed{\nabla\cdot u = 0}$$

from which

$$\int_S u_n\, dS = 0.$$

Poynting's theorem, being a consequence of the general equations, doesn't add anything to them. The localization of energy

---

[1]See the article of M. Voss in Encyklop. d. math. Wissenschaften, vol. IV, art. 1, 1901, p. 111 - 114.

must therefore be attributed to a number of logically useless (and maybe harmful at times) *conceptions in the theory*.

But there is another aspect for which its important to consider 344 Poynting's theorem.

The primary source of the conservation of energy theorem has been, and will continue to be, the experimental impossibility of *perpetual motion*, an impossibility that must exist, whatever our ideas may be on the portions of energy that the ether is obliged to supply in the absense of material bodies. The energy theorem, in its classical form

$$W = \text{Const.},$$

explained this impossibility. Poynting's theorem, in demanding only the possibility of the transformation of an integral of volume (already partly arbitrary) to an integral of surface, expresses a lot less. *Far from giving an account for this impossibility, it* 344 *readily allows the creation of a* perpetual motion. This is to say that, as long as we have not introduced the hypothesis of retarded potentials, a continuous portion of the energy, from converging waves coming from infinity, remains just as possible as the lost energy that we observe in reality. If an engine could perpetually draw energy from ether solely, independently of the presence of material bodies, it could have *perpetual motion*. We know therefore, that in adopting the formula for retarded potentials, we must show[1] that an *accelerated* particle *loses* energy and undergoes, as a result, a reaction proportional to the derivative of the acceleration. We only have to change the sign of c to pass to the hypothesis of convergent waves. We see then that the sign of the radiant vector changes also, and

---

[1] LORENTZ, *Elektronentheorie*, p. 186 - LARMOR, *Aether and Matter*, Chap. XIV.

the new hypothesis will lead, for example in the case of a vibrating particle, to a continuous increase in the amplitude with time, and more generally to an increase of energy in the system.

Poynting's theorem expresses the law of energy only when we replace the fields with their representations based on retarded potentials, a restriction which removes a lot of its elegance and scope.

If we begin from a state in which radiation is sensibly nil and where the energy E_1 can be converted into work, and we arrive at an analogous state (energy E_2), the system supposedly isolated from the 345
action of external bodies, we will have, according to Lorentz's theory, (which supposes the admissibility of the formula for potentials)

$$E_1 \geqq E_2$$

the equality taking place only if radiation was continuously nil. 345
The impossibility of perpetual motion also produces, in an essentially irreversible system, only the inequality: the energy can never increase. There is a parallelism, in this relation, with the law of entropy. In fact, electromagnetic energy is not conserved in general. This is to say that W = const. doesn't exist. We save the law of conservation of energy by attributing to the ether the lost quantity, and this procedure has decidedly great advantages, especially when we can completely recover the energy lost by the system by means of bodies that don't sensibly exert action on it, like the black bodies in Optics. But, with this energy not producing, in this hypothetical setting, any modifications which would be perceptible to our senses, we can ask ourselves if, under these conditions, would it not be possible to save likewise all other similar laws, as we have effectively done for electromagnetic momentum.

In the most general case of electromagnetic radiation, conservation of energy is no longer a law, but a convention. This is a fairly frequent development in the domain of physical truths, as Poincaré[1] stated.

§ 5. - GRAVITATION

If we consider the electromagnetic theories in their present form as a general basis for the explanation of physical phenomena, a role that only Mechanics played until now, it will suffice, at first, to ask ourselves if we can place gravitation in this general scheme. Is the notion of field, with its consequences, capable of being applied? 346
The response given by Maxwell[2] is negative.

In introducing the force R_x R_y R_z which gravitation exercises at a point xyz in space on a unit of mass, we can well, as in elec- 346
trostatics, determine this force by the system of equations (μ = density)

$$\frac{\partial R_x}{\partial x} + \frac{\partial R_y}{\partial y} + \frac{\partial R_z}{\partial z} = -4\pi\mu,$$

$$\frac{\partial R_y}{\partial z} - \frac{\partial R_z}{\partial y} = 0, \quad \frac{\partial R_z}{\partial x} - \frac{\partial R_x}{\partial z} = 0, \quad \frac{\partial R_x}{\partial y} - \frac{\partial R_y}{\partial x} = 0, \quad R_\infty = 0,$$

and the value of the energy will be

$$E = -\frac{1}{8\pi}\int d\tau\left(R_x^2 + R_y^2 + R_z^2\right) + \text{const.}$$

(the unit mass being chosen properly). Since there is attraction, the integral has the sign (-). But, says Maxwell,

---

[1] H. POINCARÉ, Science et Hypothese. Paris, 1901. Says in closing that the statement "the energy of the whole world conserves itself" does not make any sense, save for a space having a positive curvature.

[2] MAXWELL, Scient. Papers, volume 1, p. 570.

the energy being essentially positive, for E to be positive, it will be necessary to choose an enormous value for the constant, greater than the greatest value that the integral could attain for all possible positions of the bodies; the intrinsic energy of the gravitational field is constrained to *decrease* wherever gravity is sensible. "Since it is impossible for me to understand how a medium could possess such properties, I cannot pursue research, in this vein, into the cause of gravitation."

We can again say that the condition of stability of a continuous medium, elastic or otherwise, is always such that the energy is minimum when the deformation is zero; here, it is maximum for R=0; *the gravitational field would be in unstable equilibrium at infinity and wherever R is zero*.

The notion of field doesn't seem applicable to gravitation; it 346
shouldn't therefore be an issue to consider as a general base for the explanation of physical phenomena.

On the contrary, the law of elementary action which results from
Lorentz's theory, if we replace electric charges by masses, can, as 347
in the similar laws of Weber, Gauss, etc., replace the classical law of gravitation, without the new terms and the propagation they introduce having an appreciable influence on astronomical phenomena[1]; these terms are in fact second order[2] and therefore extremely small. We know that Laplace had arrived at the

---

[1]LORENTZ, *Zittings verslag*, Amsterdam, t. VIII, 1900, p. 603 -- WILKENS, *Physik. Zeitschr.*, t. VII, 1906. p. 846. -- WALKER, *Physik. Zeitshchr.*, t. VII, 1906, p. 300.

[2]This is to say that the factor $1/C = 10^{-10}/3$ is of the second order

result that the propagation speed of gravitation is at least 100,000,000 times greater than that of light; but this is related to that which, in his manner of conceiving propagation, introduces a first order term, and that moreover, this term corresponds to a friction, which doesn't happen with Lorentz.

Zöllner's explanation adopted by Lorentz is, as we know, that the attraction of two electric charges of opposite sign, is slightly greater that the repulsion of two charges of like sign and of the same absolute value. This explanation destroys the unity of the electric field, and is thus applicable only to elementary actions.

§ 6. - ACTION AND REACTION

With the ether acting on ions without undergoing action itself, Newton's principle is not satisfied by Lorentz's theory, and Poincaré[1] 347
has shown that we have for the resultant of translation

$$\int_{\infty} \frac{d\tau}{c^2} S_x, \quad \int_{\infty} \frac{d\tau}{c^2} S_y, \quad \int_{\infty} \frac{d\tau}{c^2} S_z,$$

where the integrals are extended over all space and S is the radiant vector. Furthermore, an electrified body in uniform motion exerts on itself, in general, a couple. It is important to consider separately the diverse aspects of the question which this poses: can we, from the view-point of <u>the facts</u>, draw from this inequality of action and reaction an objection to Lorentz's theory? The answer is affirmative.

348

Lets consider, at first, two electrons with charges e, e', with coordinates xyz, $x'y'z'$, velocities v, v' and accelerations

---

[1] Archives néerl., 2e serie, t. V, 1900, p. 252: Electricite et Optique, p. 448.

w, w', placed a great distance apart relative to their dimensions. Liénard[1] and Wiechert[2] have shown that for the potentials produced by e' we have

$$\Phi = \frac{e'}{\left[r\left(1-\frac{v'_r}{c}\right)\right]}, \qquad A_x = \frac{e'[v'_x]}{\left[r\left(1-\frac{v'_r}{c}\right)\right]}, \quad A_y = \ldots,$$

where we have to take the quantities in brackets at a previous instant $t - r/c$ such that the wave emitted at this instant reaches (xyz) at t; the vector r is directed from e' towards e, and we have the equation

$$r^2 = \left[x(t) - x'\left(t-\frac{r}{c}\right)\right]^2 + \left[y(t) - y'\left(t-\frac{r}{c}\right)\right]^2 + \left[z(t) - z'\left(t-\frac{r}{c}\right)\right]^2.$$

It will suffice to consider the particular case where the velocities and accelerations are small, so that we can set 348

$$x'\left(t-\frac{r}{c}\right) = x'(t) - \frac{r}{c}v'_x(t) + \frac{r^2}{2c^2}w'_x(t).$$

A simple calculation, that we will furthermore find in the Second Part, leads to the development of Φ and A, from which comes, for the force F_x exerted by e' on e, the expression

$$F_x = ee'\left[\frac{\cos\rho x}{\rho^2}\left(1 + \frac{v'^2 - 3v'^2_\rho}{2c^2} - \frac{v_x v'_x + v_y v'_y + v_z v'_z}{c^2}\right) + \frac{v'_x v_\rho}{\rho^2 c^2} - \frac{w'_x + w'_\rho \cos\rho x}{2c^2\rho}\right], \tag{13}$$

where all the quantities v', w', v must be taken at

---

[1]LIENARD, L'éclairage électrique, t. XVI, 1898, p. 5, 53, 106.

[2]WIECHERT, Archives neerl., 2e serie, t. V, 1900, p. 549.

the instant t and where ρ is the actual distance between the points e, e'.

This expression contains velocities and accelerations in a non-symmetrical manner that clearly shows the inequality of action and reaction, even when the accelerations are supposedly negligible and there is no radiation. In the case of uniform translational motion 349
of the points we have

$$v = v', \qquad w' = 0;$$

the term which is multiplied by $\cos(\rho, x)$ is directed along the line of junction ee' and satisfies the principle; the term $\frac{V_x V_\rho}{\rho^2 c^2}$ gives a force parallel to v, applied to e, and another one, equal and opposite, applied to e'. If the charges e, e' were connected by a rigid member, these two forces would produce a couple whose axis would be perpendicular to the velocity v and the junction line ρ. 349

In the Second Part we will see that no experiment requires this dissymmetry where the velocities are concerned, and that it is evident a priori. Since no experiment has shown anything but relative motions, expression (13) must be replaceable by another, of the second degree, which contains relative velocities only. Such an expression, constrained to be a vector component, wouldn't present such a dissymmetry.

On the other hand, one experiment by Trouton and Noble[1], which should have, for the case of a charged condenser, shown evidence of the couple under question, gave a negative result. In that it concerns the terms relative to velocities, the inequality of action and reaction constitutes therefore a serious objection to Lorentz's theory.

We can't say very much, from an experimental point of view

---

[1] London Transact., A, t. CCII, 1903, p. 165.

about the non symmetrical terms which are dependent on the acceleration **w'**. They contribute, even at small velocities, and when certain conditions of symmetry are satisfied, to electromagnetic mass and more generally to a reaction of inertia. For a uniformly charged sphere, of radius R, the result of elementary actions

$$\frac{de\,de'}{2c^2\rho}\left(w'_x + w'_\rho \cos \rho x\right)$$

is

$$\frac{4}{5}\frac{e^2}{R}w_x\,,\quad \frac{4}{5}\frac{e^2}{R}w_y\,,\quad \frac{4}{5}\frac{e^2}{R}w_z\,;$$

the quantity $\frac{4}{5}\frac{e^2}{R}$ is therefore the electromagnetic mass and even 350
in excluding Kaufmann's experiments nothing permits denying the possibility of such a reaction. On the contrary, it is evident that there's a considerable advantage, from the point of view of unifying our concepts, to be able to deduce the reaction of inertia and kinetic energy from electromagnetic energy. We will further 350
study the question of the variability of mass as a function of velocity.

Hertz's theory satisfies the principle in a general manner. For example, with the pressure that light exerts on a body immersed in dielectric air or ether there corresponds a reaction of the same magnitude applied to these dielectrics[1], in such a way that, in the first case, the principle is satisfied by considering the [air] medium only. But experiment has shown the existence of this pressure, even in the most perfect vacumn. In this latter case there is no reaction according to Lorentz, but, according to Hertz, there really is one, and the ether is set into motion. However, to make this perceptible, the ether would have to quit

[1]POINCARÉ, loc. cit.

concealing itself in all the experiments. Since it doesn't respect this wish, it is difficult to say if, in this case, whether the logical advantage is on Lorentz's side, who simply expresses the idea of action without reaction, or whether it is on Hertz's side who saves the principle, but in such a manner that it becomes a simple agreement.

If we are content with the forces exerted by ions on one another existing without the intervention of an intermediate, such as ether, then the finite speed of propagation leads to the lack of simultaniety and to the inequality of actions of ions on one another when they are separated (generally at least).

In the classical theories of Optics, for example in Sellmeier and Helmholtz's dispersion theory, the action of light on molecules is equal to the reaction of the molecules on the ether. The prin-
ciple was never considered as being applicable solely to the material. 350
What we can object to, in the theory, is that it would be more satisfactory if the intermediate were devised in such a manner as to explain the matter of the equality of action and reaction, and I indicated in the introduction that radiant energy materializing and projecting at the speed of light constituted such an intermediate[1].
We return therefore, in a new form, to the emission theory [of light], and to use 351
Poincaré's example, the recoil of an artillery piece and the force experienced by a body that transmits a wave of radiant energy in a certain direction are absolutely analagous, which is not the case when, instead of using this model, we consider the energy to be propagated (the ether theory).

Poincaré has shown that the inequality of action and

---

[1]It also permits avoiding absolute motion and the other difficulties (<u>see</u> Introduction and Second Part).

reaction doesn't lead to perpetual motion in Lorentz's theory; additionally, under these conditions, we are obliged to admit the hypothesis of retarded potentials.

§ 7.—Analogy Between Ether and Elastic Bodies

Maxwell's and Lorentz's equations take, in the case of pure ether, a form remarkably analogous to the ones for the equations of elasticity. What is the real significance of this analogy?

The electric vector E is satisfied, in ether, by the equations

$$(14)\quad \begin{cases} \dfrac{1}{c^2}\dfrac{\partial^2 E_x}{\partial t^2} - \Delta E_x = 0, \quad \dfrac{1}{c^2}\dfrac{\partial^2 E_y}{\partial t^2} - \Delta E_y = 0, \quad \ldots, \\ \dfrac{\partial E_x}{\partial x} + \dfrac{\partial E_y}{\partial y} + \dfrac{\partial E_z}{\partial z} = 0, \end{cases}$$

and likewise for H. This is the immediate consequence of the fundamental equations (I) through (IV).

On the other hand, the components of displacement, ξ, η, ζ, assumed to be small with respect to a point in an elastic body, 351
A and B being constants, and ρ the density; we have

$$(15)\quad \begin{cases} \rho\dfrac{\partial^2 \xi}{\partial t^2} = A\,\Delta\xi + B\dfrac{\partial\sigma}{\partial x}, \\ \rho\dfrac{\partial^2 \eta}{\partial t^2} = A\,\Delta\eta + B\dfrac{\partial\sigma}{\partial y}, \\ \rho\dfrac{\partial^2 \zeta}{\partial t^2} = A\,\Delta\zeta + B\dfrac{\partial\sigma}{\partial z} \\ \text{where}\quad \sigma = \dfrac{\partial\xi}{\partial x} + \dfrac{\partial\eta}{\partial y} + \dfrac{\partial\zeta}{\partial z}. \end{cases}$$

352

Electromagnetic theory shows, as we know, that E is identical to Fresnel's vector, H to Neuman's vector (parallel to the plane of polarization). This identity with systems (14) and (15) leads to an elastic theory of light. To do that, we have to admit either the incompressibility of ether, that is to say the condition

$$\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} + \frac{\partial \zeta}{\partial z} = 0$$

or the condition A + B = 0. In both cases, the identification is immediate. These two ways of explaining the non-existence of longitudinal waves have been admitted. In each of these hypotheses we could again choose between Fresnel's theory which leads to identifying the speed

$$\left(\frac{\partial \xi}{\partial t}, \frac{\partial \eta}{\partial t}, \frac{\partial \zeta}{\partial t} \right)$$

of ether with E, or Neumann's theory which replaces E with H.

What are the general conditions, necessary and sufficient, such that a physical phenomenon, characterized by a vector, will follow the laws expressed by (15)? I say that they are the following:

1. The phenomenon is reversible.
2. ξ, η, ζ satisfy a system of three partial differential equa- 352
tions which are of second order at most, and at least are linear to a first approximation.
3. The medium is isotropic and homogeneous.

Indeed, in considering reversibility, the equations don't have first derivatives with respect to time; we will be able to solve them by means of their second derivatives

$$\frac{\partial^2 \xi}{\partial t^2}, \frac{\partial^2 \eta}{\partial t^2}, \frac{\partial^2 \zeta}{\partial t^2}$$

which are vector components. Considering homogeniety, the right hand terms will have constant coefficients, and considering isotropy they will be the summations of vector components that were obtained by differentiation of ξ, η, ζ with respect

to x,y,z. But Burkhardt has determined all of these vectors. When 353
we admit only the first and second derivatives, only three exist
which are linearly independent, namely

$$\left(\frac{\partial\eta}{\partial z}-\frac{\partial\zeta}{\partial y},\ \frac{\partial\zeta}{\partial x}-\frac{\partial\xi}{\partial z},\ \frac{\partial\xi}{\partial y}-\frac{\partial\eta}{\partial x}\right),$$

$$(\Delta\xi,\ \Delta\eta,\ \Delta\zeta),$$

$$\left(\frac{\partial\sigma}{\partial x},\ \frac{\partial\sigma}{\partial y},\frac{\partial\sigma}{\partial z}\right),$$

and we will have, a,b,c being constants,

$$\frac{\partial^2\xi}{\partial t^2}=a\left(\frac{\partial\eta}{\partial z}-\frac{\partial\zeta}{\partial y}\right)+b\Delta\xi+c\frac{\partial\sigma}{\partial x},$$

$$\frac{\partial^2\eta}{\partial t^2}=a\left(\frac{\partial\eta}{\partial x}-\frac{\partial\xi}{\partial z}\right)+b\Delta\eta+c\frac{\partial\sigma}{\partial y},$$

$$\frac{\partial^2\zeta}{\partial t^2}=\ldots\ldots$$

In changing the signs of x,y,z, and keeping in mind the complete isotropy, we find a = 0. Therefore only system (15) remains, Q. E. D.

353

The condition[1] is satisfied by all mechanical phenomena (and we have seen that in reality it shouldn't be satisfied by electromagnetic equations which correspond to irreversible phenomena); (2) and (3) are satisfied by the phenomena of diffusion, heat propagation, and others which certainly don't present any natural connection between them. We will conclude that the analogy between (14) and (15) is a lot less characteristic than we would be inclined to believe at first. We won't conclude that there's any real physical connection between the two orders of phenomena unless the analogy carrys over even beyond this general analytical

[1]BURKHARDT, Math. Annalen, t. XLIII, 1893, p. 197; Enzyclop. d. math. Wiss., Bd. IV, Art. 14, 1901, p. 20.

form. But this is precisely not the case. Indeed, the hypothesis of the nil speed of propagation for longitudinal waves (A + B = 0) is not, as Green and Cauchy have previously said, admissible for a finite elastic body; such a body would not offer any resistance to compression, its equilibrium would be unstable. It is only recently that Lord Kelvin's gyrostatic ether has permitted us to devise such systems. On the other hand, the hypothesis of incompressibility 354
calls for the introduction to the equations one of Lagrange's factors, performing the role of pressure; the identification is no longer possible in the cases where the pressure is constant. Finally, the limiting conditions demanded by Optics are not the same as in the theory of elasticity.

I don't believe therefore that we should consider a priori *these analogies as indicative of profound physical agreement between the two domains*. If we adopt this conclusion, we won't be overly amazed 354
at the difficulties and strangeness that accompany all attempts made to extend these analogies of pure ether (where Maxwell's equations express only the fact of uniform propagation) to reciprocal actions of electric charges and the ether, expressed in the general equations (I) thru (VI). For this part of the question, I can do no more than return to the chapter which Poincare has dedicated in his Lesons[1] to the most remarkable, it seems, of these attempts: that of Larmor.

§ 8.- ELECTRODYNAMIC MASS

The remarkable experiments of Kaufmann on the electric and magnetic deviability of β rays from radium

---

[1] *Electricite et Optique*, 2e ed., p. 577 and following. -- LARMOR, *Aether and Matter*, Cambridge, 1900.

have lead to the belief that the mass of corpuscles or electrons depends on their velocity and is entirely of electromagnetic origin. The existence of an electric inertial reaction and its variability with velocity was anticipated by the theory which at a first glance seems to have received a remarkable confirmation. However, in view of the scope of these conclusions, it is advisable to find out if they are absolutely indispensable.

Lets recall that in these experiments, a beam of β rays is simultaneously subjected to the action of an electric field E, producing a deviation y, and to a magnetic field H parallel to E producing a deviation z, perpendicular to the first. A photographic plate, perpendicular to the non deviated rays, records an impression of the
rays, and permits a direct measurement of y and z. With m and v the 355
mass and velocity of an electron, e its charge, and a and b two constants of the apparatus; we have, according to Lorentz's theory

$$(16) \qquad y = \frac{eEa}{mv^2}, \qquad z = \frac{eHb}{mv}.$$

Radium, within certain limits, emits rays of all velocities. These equations, where v plays a parametric role, represent a certain parabola if m doesn't depend on v. But, the curve observed by Kaufmann is different. This occurence calls for various explanations.

1° According to Lorentz, the movement of an electrified body <u>with respect to the ether</u> is equivalent to an electric current whose field, by an effect analogous to self induction, reacts on the body and produces a force which, in the most general hypotheses, is a linear function of the acceleration components, the coefficients (for transverse and longitudinal mass) being functions determined by <u>absolute</u> velocity v, known, for example, for the sphere and ellipsoid. In the case of Kaufmann's experiments, only the transverse mass $\mu(\underline{v})$ comes into play. If we use the function $\mu(\underline{v})$ in (16) instead of $\underline{m}$, we very closely approximate the observed curve, whether we consider the electron as a rigid sphere (Abraham) or whether we only consider its volume to be constant (Bucherer and Langevin).

To appreciate the value of this interpretation lets recall that the calculation of electrodynamic mass relies uniquely on the motion of the electrified body with respect to the ether; the position and velocity of other bodies being of no consequence. It is the body's <u>absolute velocity</u> which is used in the formula. Under this interpretation, Kaufmann's experiment will be therefore the first to present evidence of an absolute motion. Now, regarding this delicate question, Lorentz's theory, at least in the form as expressed in section 1, is in disaccord with the experiment, and this disaccord 355
carries over in particular to the expression of electromagnetic momentum G [1], from which Abraham has deduced values for longi- 356
tudinal and transverse masses. Indeed, from the calculation of G we deduce[2], for the case of a charged condenser carried along by the earth's translational motion, the existence of a second order couple impressed on the condenser. But, Trouton and Noble, who did this experiment[3], didn't observe the couple. Therefore the quantity G doesn't depend on the absolute velocity or, at least not in the same manner as in Lorentz's theory. We have to conclude <u>that, even if the agreement between Abraham's theory and Kaufmann's experiments were perfect, this theory should none-the-less be considered as doubtful</u>.

---

[1] The vector is, as we know, the integral of the radiant vector extended over all space, multiplied by $1 / C^2$.

[2] H.A. LORENTZ, <u>Elecktronetheorie</u>, p. 257.

[3] London Trans., A. t. CCII. 1903. p. 165.

2° In striving to eliminate the influence of absolute motion in his equations, Lorentz was led to certain new hypotheses, to which I will return in the next paragraph. The dimensions of electrons, in particular, would be reduced to $(1 - \frac{p^2}{c^2})^{-\frac{1}{2}}$ of their value when animated by the absolute velocity p. This hypothesis leads to new formulae for mass, that Kaufmann considered as irreconcilable with his last experiments.[1] But this conclusion seems doubtful to me. Indeed, lets take for H, E, a, b the directly observed values: where instead of the

$$\frac{e}{m_0} = 1.880 \times 10^7$$

value which corresponds to cathode rays, lets take, in the formula of Langevin and Bucherer

$$\frac{e}{m_0} = 1.955 \times 10^7,$$ 356

in that of Abraham

$$\frac{e}{m_0} = 2.010 \times 10^7,$$

and finally in that of Lorentz 357

$$\frac{e}{m_0} = 2.125 \times 10^7.$$

This means to multiply in the ratios of 1 to 1.040, 1.070, and 1.130, the abscissae and the ordinates calculated by Kaufmann for $\frac{e}{m_0} = 1.880 \times 10^7$ (*loc. cit.*, p. 534). We thus find three curves for which the errors are on the order of the experimental errors, as we verified in displaying

---

[1] *Annalen der Physik*, t.XIX, 1906, p. 487; *see* also M. PLANCK, *Physik. Zeitschr.*, t. VII, 1906, p. 754.

these values on the curve, fig. 11, of Kaufmann. That savant has done direct observations and then, using the method of least squares, deduced from the various proposed formulae, a certain constant E/M_c; independent of the hypotheses developed on $\frac{e}{m_o}$; Lorentz's formula giving an inadmissable value. Our calculation shows that the value of E/M_c, determined in this manner, involves a considerable uncertanty, for we took the directly observed value, and the errors that result don't exceed the admissible magnitude. As to the value $e/m_o = 1.880 \times 10^7$, its application to β rays of radium can't be permitted, since it is not, in general, applicable for the Zeeman effect.

In short, these observations don't therefore permit the preference of any one of these formulae over another, and it would be easy to expand on this more so. 357

But is is important to observe that Lorentz's new hypotheses involve a modification in the expression for the force that two moving electrified bodies exert on one another, a modification which, as shown in a simple argument, is only appreciable for velocities comparable to that of light. That is to say, for Kaufmann's experiment only.

3° This leads to a general remark. It is easy to introduce the terms presenting this peculiarity into the equations of Electrodynamics. Since the system of equations (I) to (IV) can be replaced by elementary actions, it suffices to consider them here. Now the force 358
exerted by the particle e', having velocity v', on the particle e, having velocity v, is expressed in this latter approach in a linear form with the factor $1/c^2$; it is expressed in the former for the case of uniform motion, in a very complicated form

(given by Schwarzschild); we have seen, in section 6, the first terms of the expansion for when v'/c is small. The dissymmetry established thusly between v and v' is not confirmed by experiment, and there are, as we will see in the Second Part, an infinity of formulae, containing only relative velocities and consequently differing from that of Lorentz by the terms in $1/c^2$. For a better reason, we can add the terms containing v/c, of the third and fourth orders, without which the formulae fail to be in accord with all the experiments when v/c is small. Lorentz's elementary actions formula can be only the beginning of a serial development. We will be able to dispose of the arbitrary functions of v thus introduced in (16) in order to produce agreement with Kaufmann's experiments by the hypothesis of constant mass, and in a manner so as to 358
completely safeguard the relativity of motion. This is what will be shown in greater detail in the Second Part of this work.

In summary, instead of a mass becoming infinite when we approach the speed of light, we would have forces that are annulled because they propagate precisely with the velocity of the mobile electron.

But there is more. The very form of the curve, and the existence of a point, where the deviations y and z are null, and the velocity equal to c remain doubtful. Indeed, lets suppose

$$(17) \qquad y = \frac{e}{m}\frac{Ea}{v^2}, \qquad z = \frac{e}{m}\frac{Hb}{v}\sqrt{1-\left(\frac{v}{2c}\right)^2}, \qquad m = \text{const.},$$

from whence the parabola

$$\frac{z^2}{H^2 b^2} = \frac{e}{Eam}\,y - \frac{e^2}{4c^2 m^2},$$

whereas, according to Lorentz, for m = const. we have the comparable parabola

$$\frac{z^2}{H^2 b^2} = \frac{e}{Eam} y.$$

The values of E, H, a, b being those given by Kaufmann, and 359
$e/m = 1.830 \times 10^7$, and for the refined values of y', z', Kaufmann (loc. cit., p. 529), the parabola becomes

$$y' = 0.0160 + 0.556z'^2.$$

		y' calc. - y' obs.		
z'	y' obs.	I.	II.	
0.1350	0.0246	+0.0015	+0.0014	
0.1919	0.0376	- 11	- 9	
0.2400	0.0502	- 22	- 18	
0.2890	0.0645	- 20	- 11	
0.3359	0.0811	- 20	- 7	
0.3832	0.1001	- 25	- 8	359
0.4305	0.1205	- 15	+ 10	
0.4735	0.1405	+ 2	+ 30	
0.5252	0.1667	+ 25	+ 63	

The difference between the various individual values of y' observed by Kaufmann and the curve which represents the average for his experiments is often greater than 0.0030. The third column errors, although systematic, must therefore be considered as admissible. Those in the fourth column, which correspond to the hypothesis

$$\frac{e}{m} = 1.780 \times 10^7$$

are completely explainable as experimental errors, except for the last two. But we must note that a one percent error in the absolute measurement of the magnetic field, which can very well result from an accumulation of errors that accompany the various observations necessary for this measurement, would be especially sensible in these two instances, and change their y' by two percent, that is to say, by 0.0034. These last points have been observed only two times and Kaufmann observes that we are inclined, at the extremities of the curve where the intensity is weak, to scrutinize the extension, that is to say the tangent, where there results a value too weak for y'. These two considerations would be sufficient to explain the errors in the fourth column. But, in this curve for (17), the critical aspect is not the velocity of light c, but the maximum relative velocity $2c$ of two light rays within the same fixed system, for which 360 there is nothing surprising in a theory which considers only relative velocities.

The magnetic deviation z' is nil at this point, but not the electric deviation, which is almost equal to half the width (0.03) of the curve.

We see how expansive the field is which remains for 360 hypotheses.

In closing, lets note that the velocity v simply performs a parametric role, and is determined for each point on the curve by means of observed values of y and z. The result differs according to the theory used, and we can represent a curve in an infinite number of ways for a given parameter. It would be otherwise if direct and precise experiments, such as the ones executed by Des Coudres and Wiechert through the means of Hertzian oscillations on cathode rays, could give a direct determination of v, but such experiments don't seem realizable.

Kaufmann's experiments can therefore equally be interpreted in by modifying the existing laws of Electrodynamics in a manner that eliminates absolute motion and by making the electrodynamic mass constant. We can no longer conclude from these experiments that the mass of electrons is of electromagnetic origin; but it remains possible and the unity of physical forces gains by this hypothesis. No matter which theory is used, these experiments will play a very important role.

expand

§ 9. -- Absolute Motion

In placing the hypothesis of ether at the base of Electrodynamics and Optics, we of necessity introduce, at least for the propagation of light and electric actions, a system of coordinates, independent of ordinary matter. We should therefore expect, and we have, indeed, long expected , an influence of absolute motion or motion with respect to the assumed ether. We know that the experiments have always been negative. Lorentz's theory gives this result when we consider (only) first order terms; but the experiments of Michelson 361
and Morley, Trouton and Noble, and Lord Rayleigh, which should have 361
shown second order effects, have, contrary to the theory, also given negative results. Lorentz and FitzGerald have therefore assumed that all bodies experience a contraction, of the relation $(1 - v^2/c^2)^{-\frac{1}{2}}$, in the direction of their velocity $\underline{v}$; we thus account for the observed negative effects.[1] To explain this contraction, Lorentz calls to mind that, according to his theory, for a system of electric charges S, at rest, in equilibrium, this same system, when

---

[1] M. Planck has shown that if we assume that the density of the ether at the earth's surface is less than 50,000 times greater than that in the interplanetary medium, without any appreciable change in its properties resulting, we may possible reconcile the aberration theory with the hypothesis that the ether is entrained in the earth's motion (see LORENTZ, Enzyklop. math. Wiss., t. V, art. 13;, p. 104). This would be a very strange property of the ether.

assumed to be antimated by a uniform translational motion v, will still be in equilibrium if we modify its dimensions by the indicated ratio. Therefore, if the molecular actions obey the law of electrostatic actions, and if we can exclude the molecular motion, the molecules of a solid body should of necessity take the position of equilibrium, the accepted contraction will take place.

It is evident that this hypothesis confuses our notions of solids. The invariability of certain bodies, when we transfer them from one place to another, when we change their direction or their speed, gives us the experimental definition of distance and of other geometric magnitudes. The bodies that we use, necessarily participating in the motion of the earth, will always have an infinity of movements and rotations,
which change their dimensions; and since we don't have any way to pre- 361
cisely determine the absolute motion which comes into play here, these deformations remain absolutely unknown. How do you physically define the <u>true</u> length of a body? Does the assertion of the <u>reality</u> of this contraction have any sense? It results, from the researches of Einstein, to which we return later on, that the answer is <u>negative</u>.

The question of stability brings forth a second objection. A system 362
of electric sharges, subject <u>only</u> to electrostatic forces, is never in stable equilibrium. This is evident when the sole restriction imposed is the conservation of electricity: in changing all the dimensions of the system by the ratio of 1 to $1+\varepsilon$, the charges of elements of corresponding volume being equal, we will have performed a deformation compatible with the conditions of the system; the energy will fall to $\left(\frac{1}{1+\varepsilon}\right)^{th}$ of its original value: the equilibrium, therefore, is not stable. The sphere, for example, is, for a

deformable electron, a shape of unstable equilibrium, and its the same if we suppose, with Bucherer and Langevin, that its volume is invariable [1] ; a fortiori, whereas in Lorentz's hypothesis this restriction doesn't exist. To obtain a solid body, we therefore need to add forces of a very different character from those of electrostatic forces, or of liaisons other than incompressibility, or finally of whirling motions giving a dynamic equilibrium. But in all of these cases, Lorentz's explanation no longer applies so this explanation doesn't seem acceptable.

Poincaré has finally objected to the hypothesis as being incomplete. New experiments could bring new terms into evidence and we would require new hypotheses if, as expected, results are negative. The question of the complete elimination of absolute motion was therefore posed and was addressed by Lorentz [2] , Poincaré [3] , and Einstein [4] .

It is no longer permissible to overlook the difference between "local 362
time" and "true time", which was an essential point when we were content to explain the negative results observed up until now and a few others analogous. For us to render a full account, lets consider two points A, B which move with a constant absolute velocity v in the direction AB. A luminous wave, starting from A, at instant t, will arrive at B at 363
instant t'. It will have to travel

---

[1] This is what Ehrenfest pointed out (Physikalische Zeitschriften, t. VII, 1906, p. 302): for gravitation the equilibrium would be stable; but in changing the attractions into repulsions, the energy changes sign and the equilibrium becomes unstable.

[2] Amsterdam Proceedings, 1903-1904, p. 809

[3] Comptes rendus, t. CXL, 1905, p. 1504.

[4] Ann. der Physik., t. XVII, 1905, p. 891.

the distance AB + $(t - t')v$ with the speed c; we have then

$$t' - t = \frac{AB + (t' - t)v}{c} \quad \text{or} \quad t' - t = \frac{AB}{c - v} \, .$$

The duration of transmission will depend on v, and its changes include a first order term $- AB\frac{v}{c}$, the correction (including higher order terms) being precisely the one we have to apply to the true time to get the local time. Lorentz[1] showed that for terrestrial phenomena, this correction is without influence. In particular, to determine the speed of light directly, we are obliged to make it cover a closed path which brings it back to its starting point; thus eliminating the first order terms. So, in the example considered, if the wave emitted at A is reflected at B, it will arrive at A after a time

$$t' - t = AB\left(\frac{1}{c-v} + \frac{1}{c+v}\right) = \frac{AB}{c}\left(1 + \frac{2v^2}{c^2} + \ldots\right) .$$ 363

But it will be otherwise for astronomical phenomena. In the determination of the speed of light by the occultations of satellites, we don't use a closed path, thus, the perturbation that the hypothesis of a translation of the solar system with respect to the ether would cause in the observed retardation would be of a first order and of observable magnitude.

Indeed, the delay of an occultation can reach $\frac{d}{c}$ (where d is the diameter of the terrestrial orbit), or about 1000 seconds. An absolute speed of the solar system (having no connection with the system's motion with respect to the closer fixed stars)

---

[1] Versuche einer Theorie, etc., p. 82 and following.

equal to 30 km per second in the plane of the ecliptic would bring about a correction of $1000\frac{30}{3\times10^5} = 0.1$ second for the maximum observed delay, a correction which would change sign according to the relative position of the Earth and the satellite in relation to the direction of translation. The systematic differences for a long system of obser- 364
vations can then reach 0.2 second, a quantity which is on the order of those which we observe in Astronomy[1].

If, therefore, we do not wish to admit that the speed of light depends on that of the bodies emitting it and is purely relative, like all speeds (and the *ether* concept, alone, prevents drawing out of the relativity principle this so natural consequence), *we will have to modify the definition of time*.

Lorentz has enunciated the hypotheses which would allow giving to 364
the equations for an entrained system, as well as for the axes of coordinates, in a uniform motion of translation *v* parallel to the *x* axis, the same form as for the case of rest. He admits that all the masses are function of velocity, thus abandoning the principle of the conservation of mass[2]. We must also, as we formerly revealed, suppress the notion of solid bodies and

---

[1] Supposing that the laws of gravitation are modified by motion as the laws of Electrodynamics are, the corrections would be only of the second order and could not cancel the first order term.

[2] The word *mass*, in the theories of Lorentz, Poincare and Einstein, has no precise meaning anymore: the number representing it depends on the motion of the system of coordinates, the motion being absoutely arbitrary. But the force depends also on this motion and it is not the two member of the equation

$$m\frac{d^2x}{dt^2} = x, \ldots$$

but their relation that alone remains invariable when we change this motion.

introduce a new definition of time; it will be the variable

$$t' = t\sqrt{1-\frac{v^2}{c^2}} - \frac{vx}{c^2\sqrt{1-\frac{v^2}{c^2}}}\text{[1]} \qquad \boxed{t' = \frac{t}{\beta} - \beta\frac{vx}{c^2} \quad \text{where } \beta = \frac{1}{\sqrt{1-v^2/c^2}}}$$

which will play the role of time in the equations.

Time, thus defined doesn't satisfy almost any of the axioms assigned to the notion of time in the ordinary sense. Two events, simultaneous for an observer A, but taking place at different points, are not simultaneous anymore for a second observer B [who is] in motion with respect to the first. Simultaniety becomes a relative notion.

Two equal times for observer A will not be so for B.

The parallelogram rule for velocities is only approximated: thus 365

[for] v and v' being the speeds of two bodies which are moving in opposite directions, in relation to a primary system of coordinates, the relative speed of the first [body] with respect to the second, i.e., the velocity that an observer traveling along with the second 365

[body] would observe, is not v - v', but

$$\frac{v-v'}{1+\frac{vv'}{c^2}}$$

and it will remain steadfastly inferior to the speed of light. For two β rays emitted in opposite directions by a grain of radium, each with a speed of 250,000 km per second, the relative speed will not be 500,000 km per second, but 294,000 km per second.

The words "speed", "time", etc., have therefore acquired a significance very different from that which they normally have, and now have only a relative sense.

[1] the term "x" is used here, instead of the conventional "x−vt". This may be a simple oversight.

The ether, in this new Kinematics, will not play any role because it no longer furnishes a system of absolute coordinates. But this concept will oblige us to replace the simple axioms of the conservation of mass, the invariability of solids, the parallelogram of velocities, etc., axioms which we should abandon only as a last resort, with complicated relationships presenting considerable difficulties to the imagination (similar to those of curved space in three dimensions), and which we generally can't treat *rigorously*, except by analytical considerations. We must add that this theory was presented by Lorentz with all reservations.

Einstein (*loc. cit.*) has presented the same results in a different form. He admits, *a priori*, for the speed of light, a law which *by its nature involves a large amount of arbitrariness*; the comparison with that which we will adopt in the Second Part of this Work will demon- 365
strate this sufficiently. It leads, with the principle of relativity, to a definition of the *simultaniety* of two events at two different points, of which he makes a relative notion, and more generally [leads] to the new Kinematics which has just been discussed. The simultaniety 366
included in the definition of the length of a body in motion in relation to the fixed marks of a standard measure (since it will be a matter of pointing *simultaneously* to the two extremities of the body, otherwise the body would move during the interval) this body *will appear* to be of a different length to an observer at rest, according to its more or less large speed (although its true length remains invariable). We therefore avoid the contractions admitted by Lorentz, or rather, we see that their *reality* is only a question of definition.

Einstein verifies that Lorentz's equations are thus made independent of absolute motion, and that the law admitted by him for the propagation of light is in accordance with the equations. These then, in the measure that they express this law of propagation, become superfluous; moreover, the reasoning does not demonstrate at all, as some authors have beleived, that these transformations are the only group which leaves Lorentz's equations invariable. This problem has rather to do with Poincare's methods (loc. cit.).

Bucherer[1] was led, by considerations on the relativity of motions, to abandon the notion of ether. Lorentz's equations should always be applied by assuming that the system of coordinates is at rest with respect to the point P whose motion we study. Bucherer only considers the case of uniform motions; the action on an electron propelled with a relative velocity $\underline{u} = \underline{v}' - \underline{v}$ in relation to P, according to formula
(13) (where we set $\underline{v}' = \underline{u}$, $\underline{v} = 0$), will be given by the formula 366

$$F_x = \frac{ee'\cos\rho x}{\rho^2}\left(1 + \frac{u^2 - 3u_\rho^2}{2c^2}\right), \quad F_y = \ldots.$$

For two closed currents, we verify easisly (see Second Part) that only the terms proportional to $\underline{vv}'$ play a role, with the accelerations excluded, which we haven't taken into consideration, and terms in $\underline{v}^2$, $\underline{v}'^2$; it is moreover what should happen if the intensity of the force is proportional to the product of the intensities of the currents. The action of two elements of current on each other will be from then on

$$\frac{ii'\,ds\,ds'}{\rho^2}\cos(\rho x)\left[-\cos(ds\,ds') + 3\cos(\rho\,ds)\cos(\rho\,ds')\right].$$

The hypotheses of Ampere are verified: the action is 367

[1]Physik. Zeitschr., t. VII, 1906, p. 553.

parallel to the line of junction ρ of the elements; but the parenthesis should be

$$-2\cos(ds\,ds') + 3\cos(\rho\,ds)\cos(\rho\,ds').$$

Bucherer's hypothesis is therefore irreconcilable with the laws of closed currents.

This is because the notion of field ceases to be applicable there.

We are therefore brought back to the complicated hypotheses which were set forth. We must say, in closing, that these complications occur not only at great speeds, but also in Fizeau's experiment on the entrainment of waves, for example. In effect, according to the principle of relativity, an observer carried along in the translational movement of a transparent body will find, for the speed of propagation of waves in this environment, the same value as if he were at rest (supposing the period to be the same, or the dispersion negligible). We 367
would conclude that, in ordinary Kinematics, the waves are totally carried along by the substance. There is nothing to it: the term speed has a new meaning, and in reality, Lorentz's proof continues to be applicable; we come back to Fresnel's coefficient.

§ 10 - Summary and Conclusions

We know that ether was, at first, only one of the numerous fluids of Physics, but with the new experiments having proved to Fresnel that light waves are transversal, we had to create a body analogous to elastic solids. But then, how can the other bodies move through it without experiencing any appreciable resistance? The question was all the more difficult with the problem of aberration obliging us to

admit that the ether does not participate in the earth's translational motion, so that all bodies are constantly traversed by a current of ether of 30 km per second, with zero effects, despite the rigidity of ether. We must add that the elasticity of this body is quite singular, seeing that its resistance to compression would be zero, which wouldn't happen for a finite solid. We can, it is true, appeal to Lord Kelvin's rotational elasticity, by being careful not to take into account the 368
perturbation that would be brought to this ingenious mechanism by the brutal passage of an animated body with a speed of 30 km per second.

The difficulties increased when, the identity of light and electrical oscillations having been shown, we had to extend this system of explanations to electromagnetism as a whole. Poincaré[1] has exposed some of the strangeness to which we are led. Besides, experiments have refused to accord to ether that primordial property of bodies: movement. 368
Fizeau's experiments (interpreted by Lorentz), and those of Lodge, and others, have concurred in their negative results: ether is not entrained by the movement of matter, nor by that of charged or magnetized bodies, nor by currents, etc. The hypothesis itself, of such motions, does not permit obtaining a mechanical explanation of Electrodynamics. We resigned ourselves to accept the absolute rest of ether; the hypothesis of a complete compenetration permitted the avoidance of the difficulty relative to the movement of bodies through ether. This latter has become what Drude calls a "Physical Space", the seat of electric and magnetic energy, and polarizations. It furnishes a system of coordinates independent of all

---

[1]Electricite et Optique, 2^e ed.; A propos de la theorie de Larmor, p. 577 et suiv.

matter, and to which Maxwell's and Lorentz's equations must be referred.

This is already too much abstraction. That still is not enough. Indeed, according to these views, ether could also be the seat of phenomena independent of matter, and thus manifest its existence. Nothing of the kind; and, in order to explain it, we needed a new hypothesis, discarding all waves that don't diverge from a material element of volume. The role of ether is again reduced. We have seen that, from now on, we can completely set aside the notion of field and the consideration of what is passing in the ether, and to be content with elementary actions of charges on each other (exactly as in the older theories of Gauss, Weber, Riemann and Clausius, but with a finite time of transmission). We thus express the same facts, but 368
by including the hypothesis on the divergence of waves and the consequential ireversibility, and which the equations of partial derivatives 369
are powerless to express. The ether has become a system of absolute coordinates, a mathematical abstraction; the equations of partial derivatives, an intermediate mental construct which, however, isn't sufficient in itself.

Finally, this phantom of ether itself did not stand up under the scrutiny of experiment. It seems well accepted that absolute motion cannot be put into evidence. We have seen to which hypotheses, disrupting all the principles of physics, we must have recourse to, to be aware of this result. The only conclusion which, from then on, seems possible to me, is that <u>ether doesn't exist, or more exactly, that we should renounce use of this representation; that the motion of light is a relative motion like all the others, that only relative speeds play a role in the laws of nature; and finally that we should renounce use of the equations of partial derivatives</u>

and the notion of field, in the measure that this notion introduces absolute motion.

As I already stated in the Introduction, this overly negative conclusion needs two complimentary remarks; a simple representation for the new mode of light motion; and the demonstration that a theory satisfying these principles is possible.

The habit that we have to "substantiate", if I dare to express myself so, a habit to which we owe the old caloric, magnetic, etc., fluids and the new energy fluid, makes, indeed, indispensable the introduction of a representation which makes us realize what happens to light and electrical forces when, having left a body, they don't act on still another. A theory which wouldn't admit such a representation would be considered by many as introducing actions at a distance
simply retarded. Moreover, as Poincaré noted (<u>Science et Hypothese</u>, 369
p. 199)[1], and this is one of the reasons that we can judge in favor about the existence of ether. Mechanics would have it that the state of a system depends only on the immediately preceeding states; it wouldn't be so anymore if we cancelled all intermediates. Actually, we thus save only a convention, which perhaps doesn't have any extreme usefulness. We have seen that we can't arbitrarily give the initial state
of ether, which must (should?) satisfy the formulae of (for?) retarded potentials, it (this?) 370
is to say that the consideration of the system during a finite period is not avoided effectively. On the other hand, the pressure exerted by light on a mirror, even in a vacuum, for example, is contrary to the equality of action and reaction when applied to the material alone. We will therefore have to "substantialize" the radiant energy to save the principle and that of conservation of energy whenever there is a body in which the radiation doesn't meet any material obstacle in certain

[1] This is probably page 119.

directions, and for which the energy cannot, consequently, ever be fully recovered. These principles will then become conventions, in part at least, but for a greater advantage to the economy of our thought.

SECOND PART

pages
209 - 275

(in translation) 370

NAME INDEX